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DYNAMIC MONOPOLY PRICING UNDER THE REFERENCE PRICE EFFECT

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One of the important aspects of the theory and practice of firms is development of an optimal dynamic pricing strategy. Traditional marketing models consider the consumer as a rational agent who makes decisions based on current prices. However, in the dynamics, with repeated purchases, consumers form price expectations or reference prices which are compared with current prices. This effect is known as "the reference price effect". The work focuses on the dynamic monopoly pricing in the presence of the reference price effect. To study this question, the author constructed a model of the monopoly selling the product over T periods. The study considers a case when the reference prices are formed on the basis of prices from the previous period. In this case, buyers' purchase decisions also rely upon the ratio of the current and previous prices. In the long run, the monopoly can have various efficiency criteria. The article considers two efficiency criteria: the maximum profit within each period of time and the maximum profit for the whole time. The study allowed finding discrete and equilibrium solutions. A comparative analysis of profits of the monopoly by two criteria resulted in the following conclusions. In comparison with the situation of the absence of the reference effect, optimization by the global criterion increases profits and that by the local criterion reduces them. Obtained results show that in case of optimization by the global criterion the reference effect dwindles in the long run.

Keywords: *monopoly, dynamic pricing, reference price effect*

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Мельников С. В. Динамическое ценообразование монополии с учетом эффекта справочной цены

Одним из важных вопросов теории и практики деятельности фирм является разработка оптимальной ценовой стратегии. Традиционные маркетинговые модели рассматривают потребителя как рационального агента, который принимает решения, основанные на текущих ценах. Однако в динамике при повторных покупках потребители формируют ценовые ожидания или справочные цены, которые сравнивают с текущими ценами. Данный эффект получил название «эффект справочной цены». В работе исследуется динамическое ценообразование монополии с присутствием эффекта справочной цены. С этой целью построена модель монополии, которая продает продукт в течение T периодов. Рассмотрен случай, когда справочные цены формируются на основе цен прошлого периода. В этом случае решения покупателей относительно покупки зависят также и от соотношения прошлых и текущих цен. В долгосрочном периоде монополия может иметь разные критерии эффективности. В статье рассмотрены два критерия эффективности: максимум прибыли в каждый период и максимум прибыли за все время. В ходе исследования найдены дискретные и равновесные решения. Сравнительный анализ прибылей монополии по двум критериям позволяет сделать следующие выводы. По сравнению с ситуацией отсутствия справочного эффекта оптимизация по глобальному критерию повышает прибыль, а по локальному – снижает. Определено, что при оптимизации по глобальному критерию в долгосрочном периоде справочный эффект нивелируется.

Ключевые слова: *монополия, динамическое ценообразование, эффект справочной цены*

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Мельников С. В. Динамічне ціноутворення монополії з урахуванням ефекту довідкової ціни

Одним із важливих питань теорії і практики діяльності фірм є розробка оптимальної цінової стратегії. Традиційні маркетингові моделі розглядають споживача як раціонального агента, який приймає рішення, засновані на поточних цінах. Однак у динаміці при повторних покупках споживачі формують цінові очікування або довідкові ціни, які порівнюють із поточними цінами. Даний ефект отримав назву «ефект довідкової ціни». У роботі досліджується динамічне ціноутворення монополії з присутністю ефекту довідкової ціни. З цією метою побудовано модель монополії, яка продає продукт протягом T періодів. Розглянуто випадок, коли довідкові ціни формуються на основі цін минулого періоду. В цьому випадку рішення покупців щодо покупки залежать також від співвідношення минулих і поточних цін. У довгостроковому періоді монополія може мати різні критерії ефективності. У статті розглянуто два критерії ефективності: максимум прибутку в кожен період та максимум прибутку за увесь час. У ході дослідження знайдено дискретні та рівноважні рішення. Порівняльний аналіз прибутків монополії за двома критеріями дозволяє зробити наступні висновки. Порівняно із ситуацією відсутності довідкового ефекту оптимізація по глобальному критерію підвищує прибуток, а по локальному – знижує. Визначено, що при оптимізації по глобальному критерію у довгостроковому періоді довідковий ефект нівелюється.

Ключові слова: *монополія, динамічне ціноутворення, ефект довідкової ціни*

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Introduction. One of the important aspects of the theory and practice of firms is development of an optimal dynamic pricing strategy. Traditional marketing models consider the consumer as a rational agent who makes decisions based on current prices. However, in the dynamics, with repeated purchases, consumers form price expectations or reference prices which are compared with current prices. This effect is known as "the reference price effect". Reference prices are determined by the following factors:

1. Memories of past prices. The price paid for the goods the previous time becomes a comparison basis for new prices.
2. The prices of related products and services.
3. The price of the favourite brand.
4. The nature of the industry. For example, people have come to expect large discounts in clothes shops. When they see clothes on sale at a full price, they may remember that these prices are likely to be reduced at a later stage.
5. The price which occurs in the market most often.
6. The fair price. Each consumer has their own opinion about what price level is fair.

In marketing research the effects of the reference price became widely used after the article by Monroe (1973) [7]. Putler (1992) [9] incorporates reference price effects into the traditional economic theory of consumer choice and examines the effects of reference price formation on the results of the traditional theory. Greenleaf (1995) investigates the impact of reference price effects on retail promo prices and describes why these effects can make promoting profitable. Kopalle et al. (1996) [6] consider a group of frequently purchased consumer brands which are partial substitutes and examine two situations. In the first situation a group of brands is managed by a retailer, and in the second one the brands compete in an oligopoly. Anderson et al. (2005) [1] model the temporal pricing strategies for two firms with asymmetric costs and differing market power. Fibich et al. (2005) [2] derive an analytical expression for the price elasticity of demand in the presence of reference price effects and conclude that the effect of reference price is most noticeable immediately after a price change, before consumers have had time to adjust their internal reference prices. Popescu and Wu (2007) [8] consider the dynamic pricing problem of a monopolist firm in a market with repeated interactions, where demand is sensitive to the firm's pricing history. Gavious and Lowengart (2012) [3] aim at exploring the influence of such effects on demand and study firms' pricing strategies when asymmetric effects of the reference price exist over a finite or infinite decision horizon. Kachani et al. (2014) [5] research a competitive model of duopoly market for a single product where retailers interact through their influences on the customers' reference price. Zhang (2014) [10] studies the integrated impact of the reference effect and the substitution effect on the price decisions of firms.

Thus, modeling of dynamic pricing of firms under reference price effects is certainly of theoretical and practical interest.

Problem statement. We consider a case when reference prices are formed on the basis of the prices of the previous period. In this case, the consumers' purchase decisions also depend on the ratio of the past and current prices. In the long run, a monopoly can have different efficiency criteria.

Thus the aim of this article is to research dynamic monopoly pricing for different efficiency criteria taking into account the reference price effect.

Main results and their justification. Zhang (2014) [10] investigated the model of a monopoly that sells the product in two consecutive periods. The reference price effect as an additional component of the basic demand occurs in the second period.

It is of interest to generalize the results obtained by Zhang (2014) [10] to the case of an arbitrary number of periods.

Consider a monopoly that sells a product over t t periods, $t = 0, 1, ..., T$. The set of periods where the reference price effect occurs we define as $N = \{1, 2, ..., T\}$. The product is produced at a constant unit cost z. The demand for the monopoly product depends on two factors: current price and previous price. In the zero period the effect of reference price is absent. The basic demand (in the absence of the reference price effect) is characterized by a traditional linear function $Q_0 = b - k \cdot P_0$, where P_0 and Q_0 are the price and demand volume in the zero period, *b* is market potential, and *k* is the coefficient of the price sensitivity of demand. Non-negativity condition for the demand $P_0 \le \frac{b}{k}$.

The reference price effect can be characterized as $\varphi \cdot (P_t - P_{t-1}), t \in N$, where φ is the coefficient of the reference effect measuring the consumer's sensitivity to price change in time.

Thus, starting from the first period, the consumer will receive additional gains or losses from price changes. Accordingly, the demand function will be as follows: $Q_t = b - k \cdot P_t - \varphi \cdot (P_t - P_{t-1}), t \in N$. Non-negativity condition for the demand $P_t \le \frac{b + \varphi \cdot P_{t-1}}{k + \varphi}$.

We make a natural assumption that the coefficient of price sensitivity has a greater impact on the demand than the coefficient of the reference effect, $k > \varphi$.

Consider two criteria: the maximum profit within each time period (I) and the maximum profit for the whole time (II).

Criterion I - the maximum profit within each time period. We assume that the monopolist is in the period *t* – 1 and wants to maximize profit in the next period *t*.

The monopoly's profit is

For $t = 0, \quad F_0 = (P_0 - z) \cdot (b - k \cdot P_0) \to \max_{P_0}$.

For $t \in N, \quad F_t = (P_t - z) \cdot (b - k \cdot P_t - \varphi \cdot (P_t - P_{t-1})) \to \max_{P_t}$.

By equating the first-order derivatives to zero, we find the optimal prices.

For $$t = 0, \quad P_0^{*(I)} = z + \frac{b - z \cdot k}{2 \cdot k}. \qquad (1)$$

For $$t \in N, \quad P_t^{*(I)} = z + \frac{\varphi \cdot P_{t-1} + b - z \cdot (k + \varphi)}{2 \cdot (k + \varphi)}. \qquad (2)$$

The price (1) is the optimal monopoly price in the absence of the reference effect, $P_0^{*(I)} = P_m^*$.

The expression (2) is a linear non-homogeneous first-order difference equation.

Let us present a general solution to this discrete difference equation with the initial condition $P_t = z + \frac{b - z \cdot k}{2 \cdot k}$ for $t = 0$

$$P_t^{*(I)} = z + \left(\frac{\varphi}{2 \cdot (k + \varphi)}\right)^t \cdot \frac{\varphi \cdot (b - z \cdot k)}{2 \cdot k \cdot (2 \cdot k + \varphi)} + \frac{b - z \cdot k}{2 \cdot k + \varphi}, t \in \overline{0, T}.$$

Using the equilibrium condition $P_t = P_{t-1}$, we find the equilibrium solution to the equation (2):

$$P^{e(I)} = z + \frac{b - z \cdot k}{2 \cdot k + \varphi}. \quad (3)$$

Below are the optimal trajectories of the monopoly with the initial conditions: $b = 10, k = 0{,}2, \varphi = 0{,}1, z = 6, t = \overline{0, 20}$ (Fig.1).

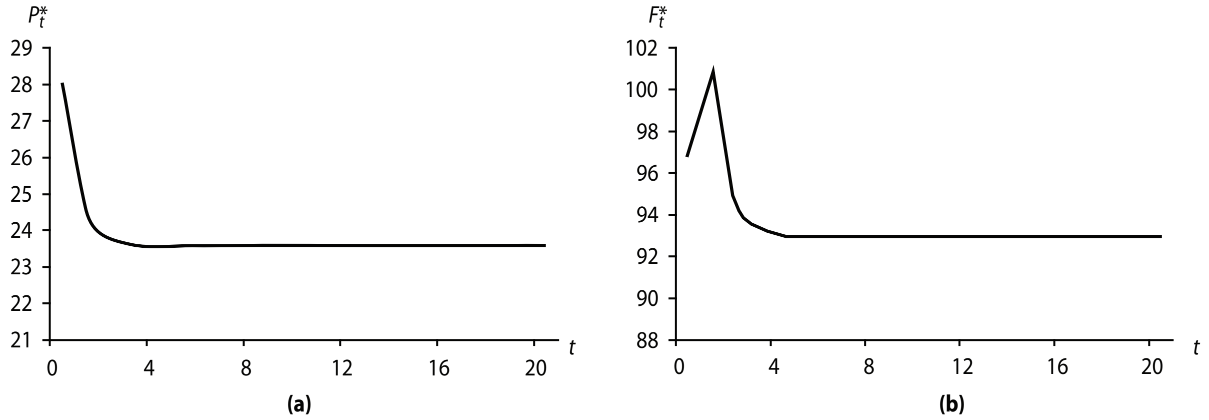


Figure 1. The trajectories of the optimal prices (a) and profit (b) by criterion I

Criterion II - the maximum profit for the whole period. The monopoly's profit is

$$F = \sum_{t=0}^{T} F_t \to \max_{\{P_t\}} \quad (4)$$

Monopoly profits within each time period.

$$\begin{cases} F_0 = (P_0 - z) \cdot (b - k \cdot P_0), \\ F_1 = (P_1 - z) \cdot (b - k \cdot P_1 - \varphi \cdot (P_1 - P_0)), \\ \dots \\ F_t = (P_t - z) \cdot (b - k \cdot P_t - \varphi \cdot (P_t - P_{t-1})), \\ F_{t+1} = (P_{t+1} - z) \cdot (b - k \cdot P_{t+1} - \varphi \cdot (P_{t+1} - P_t)), \\ \dots \\ F_T = (P_T - z) \cdot (b - k \cdot P_T - \varphi \cdot (P_T - P_{T-1})). \end{cases}$$

Equating the corresponding first-order partial derivatives to zero, we obtain a system of equations:

$$\begin{cases} P_0 = \frac{b + z \cdot (k - \varphi)}{2 \cdot k} + P_1 \cdot \frac{\varphi}{2 \cdot k}, & (5) \\ P_{t+1} - \frac{2 \cdot (k + \varphi)}{\varphi} \cdot P_t + P_{t-1} = -\frac{b + z \cdot k}{\varphi}, t \in \overline{1, T-1}, & (6) \\ P_T = \frac{b + z \cdot (k + \varphi)}{2 \cdot (k + \varphi)} + P_{T-1} \cdot \frac{\varphi}{2 \cdot (k + \varphi)}. & (7) \end{cases}$$

Let us now find a general solution to the linear non-homogeneous second-order difference equation (6).

In order to do this, we first find the solution to the homogeneous equation

$$P_{t+1} - \frac{2 \cdot (k + \varphi)}{\varphi} \cdot P_t + P_{t-1} = 0. \quad (8)$$

The characteristic equation: $\lambda^2 - \frac{2 \cdot (k + \varphi)}{\varphi} \cdot \lambda + 1 = 0$.

The discriminant equals $D = \frac{4 \cdot k \cdot (k + 2 \cdot \varphi)}{\varphi^2} > 0$.

We have two different real roots

$$\lambda_{1,2} = \frac{k + \varphi \pm \sqrt{k \cdot (k + 2 \cdot \varphi)}}{\varphi}.$$

The general solution to the equation (6) has the form $P_t^* = C_1 \cdot \lambda_1^t + C_2 \cdot \lambda_2^t + P_t^{(0)}$, where C_1, C_2 are arbitrary constants, $P_t^{(0)}$ is a particular solution to equation (6). The particular solution will be sought in the form $P_t^{(0)} = A \cdot t$.

$$A \cdot (t+1) - \frac{2 \cdot (k + \varphi)}{\varphi} \cdot A \cdot t + A \cdot (t-1) = -\frac{b + z \cdot k}{\varphi}, \quad A = \frac{b + z \cdot k}{2 \cdot k \cdot t},$$

$$P_t^{(0)} = \frac{b + z \cdot k}{2 \cdot k}. \quad (9)$$

We have obtained that a particular solution to the equation (6) coincides with the optimal monopoly price in the absence of the reference effect (1), $P_t^{(0)} = P_m^*$.

Thus, the general solution to the equation (6) is:

$$P_t^{*(II)} = C_1 \cdot \lambda_1^t + C_2 \cdot \lambda_2^t + \frac{b + z \cdot k}{2 \cdot k}. \quad (10)$$

We find arbitrary constants C_1, C_2 from the boundary conditions (5) and (7).

$$C_1 = \frac{\gamma_1 \cdot \alpha_{22} - \alpha_{12} \cdot \gamma_2}{\alpha_{11} \cdot \alpha_{22} - \alpha_{12} \cdot \alpha_{21}}, \quad C_2 = \frac{\alpha_{11} \cdot \gamma_2 - \gamma_1 \cdot \alpha_{21}}{\alpha_{11} \cdot \alpha_{22} - \alpha_{12} \cdot \alpha_{21}},$$

where

$$\alpha_{11}=1-\frac{\varphi}{2\cdot k}\cdot\lambda_1,\ \alpha_{12}=1-\frac{\varphi}{2\cdot k}\cdot\lambda_2,\ \alpha_{21}=\lambda_1^T-\frac{\varphi}{2\cdot(k+\varphi)}\cdot\lambda_1^{T-1},$$

$$\alpha_{22}=\lambda_2^T-\frac{\varphi}{2\cdot(k+\varphi)}\cdot\lambda_2^{T-1},\ \gamma_1=\frac{b+z\cdot(k-\varphi)}{2\cdot k}+\frac{b+z\cdot k}{2\cdot k}\cdot\left(\frac{\varphi}{2\cdot k}-1\right),$$

$$\gamma_2=\frac{b+z\cdot(k+\varphi)}{2\cdot(k+\varphi)}+\frac{b+z\cdot k}{2\cdot k}\cdot\left(\frac{\varphi}{2\cdot(k+\varphi)}-1\right).$$

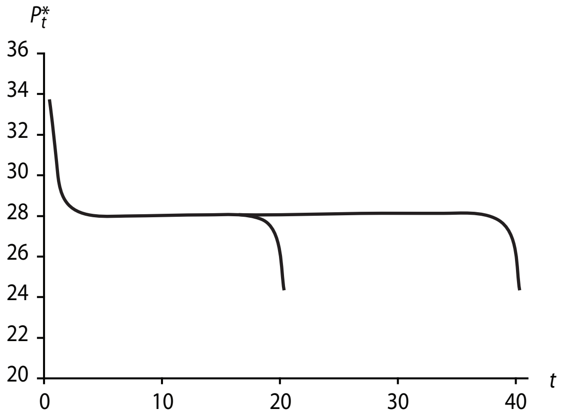
For example, for two periods $(t=0,1)$, formulas for finding the optimum prices are:

$$P_0^{*(II)}=z+(b-k\cdot z)\cdot\frac{2\cdot k+3\cdot\varphi}{4\cdot k^2+4\cdot k\cdot\varphi-\varphi^2}, \quad (11)$$

$$P_1^{*(II)}=z+(b-k\cdot z)\cdot\frac{2\cdot k+\varphi}{4\cdot k^2+4\cdot k\cdot\varphi-\varphi^2}. \quad (12)$$

At the zero unit cost $(z=0)$, formulas (11) and (12) coincide with (Zhang, 2014).

Below are the optimal trajectories of the monopoly with the initial conditions: $b=10, k=0{,}2, \varphi=0{,}1, z=6$ for $t=\overline{0,20}$ and $t=\overline{0,40}$ (Fig. 2).



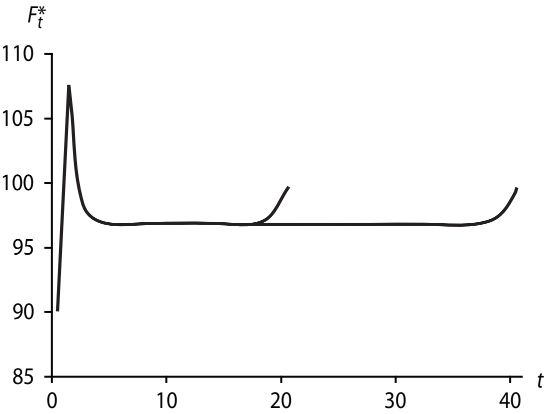


Figure 2. The trajectories of the optimal prices (a) and profit (b) by criterion II

Figure 2 shows that in the long run the optimal prices will tend to some equilibrium price. Simultaneously, the decrease in the optimal prices is connected with the end of the planning period – the boundary condition (7). Using the equilibrium condition $P_{t-1}=P_t=P_{t+1}$, we find the equilibrium solution to the equation (6):

$$P^{e(II)}=\frac{b+z\cdot k}{2\cdot k}=P_m^*.$$

Thus, in the equilibrium state $(t\to\infty)$ the impact of the reference effect on the optimal price disappears, the solution to the homogeneous equation (8) will tend to zero, and only the particular solution (9) will stand.

Zhang (2014) [10] found that $P_0^{*(II)}>P_m^*$ and $P_1^{*(II)}<P_m^*$. This result was obtained because the study considered only two periods $(t=0,1)$. Consideration of the activities of the monopoly in the long run allowed concluding that the equilibrium price will tend to the optimal price of the monopoly in the absence of the reference effect (P_m^*).

For comparison we show all optimal trajectories of the monopoly with the initial conditions: $b=10, k=0{,}2, \varphi=0{,}1,$ $z=6, t=\overline{0,20}$ (Fig. 3).

Comparing the dynamics of profit according to two criteria, we can see that the profit by the global criterion II is higher than that by the local criterion I. This is due to the fact that optimization within each period ignores all dependences of prices in time.

Conclusions. Fibich et al. (2005) [2] concluded that the effect of the reference price is most noticeable immediately after a price change, before consumers have had time to adjust their internal reference prices. This is consistent with our results. The monopoly can increase its total profit in the presence of the reference effect exactly in the initial periods (Fig. 3).

Comparative analysis of the monopoly profit by two criteria leads to the following conclusions. Compared with the situation of absence of the reference effect, the optimization by the global criterion increases profits and that by the local criterion reduces them. Also, we can conclude that in case of optimization by the global criterion in the long run the reference effect dwindles.

In the future we plan to model monopoly pricing strategies taking into account other marketing effects.

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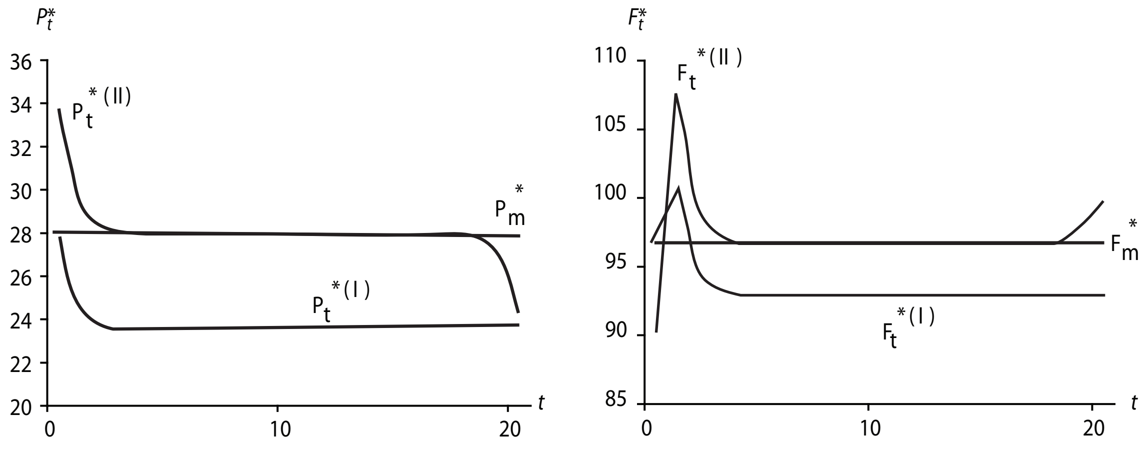

(a) (b)

Figure 3. The trajectories of the optimal prices (a) and profit (b) by all criteria

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