UDC 336.443:338.242.4.025.2
JEL Classification: C52; E52; F31

THE MATHEMATICAL MODELING OF EXCHANGE RATE DYNAMICS AS A BASIS FOR THE STATE REGULATORY POLICY*

©2025 **MALYARETS L. M., VORONIN A. V., LEBEDEVA I. L., LEBEDEV S. S.**

UDC 336.443:338.242.4.025.2
JEL Classification: C52; E52; F31

Malyarets L. M., Voronin A. V., Lebedeva I. L., Lebedev S. S.

The Mathematical Modeling of Exchange Rate Dynamics as a Basis for the State Regulatory Policy

The success of the State regulation of the currency market, which depends on the stabilization of the country's economy, support for export competitiveness, control of inflation, etc., is determined by the model used as the basis for making managerial decisions. This article proposes two approaches to solving the problem of optimal currency rate regulation. One approach is based on the use of a uniform frequency quality management criterion, while the other relies on the application of an integral quadratic quality management criterion for balancing demand and supply in the currency market. The aim of this work was to build a mathematical model that would allow determining the optimal parameters of the currency market and the conditions under which the deviation of the actual currency rate from the necessary fixed rate would be minimal. Within the proposed mathematical model, it is assumed that all processes considered are continuous in time. This allows for the application of elements of integral and differential calculus for constructing the model. The obtained mathematical model represents a system of differential equations and includes two types of quality criteria that, based on the analysis of possible fluctuations in the exchange rate, allow for the selection of methods for its regulation. However, these criteria have different orientations. Thus, the application of the uniform-frequency criterion is aimed at reducing the volatility of exchange rate deviations from the necessary value under the worst influence of seasonal fluctuations in currency demand. Meanwhile, implementing exchange rate regulation using the integral quadratic quality management criterion allows for the optimization of costs for stabilizing the exchange rate over long time intervals. The results obtained allow for using the proposed mathematical model to substantiate the choice of ways to optimize the regulatory policy of the state in the currency market.

Keywords: *uniform-frequency criterion, exchange rate, currency market, mathematical model.*

DOI: *https://doi.org/10.32983/2222-0712-2025-2-227-233*

Formulae: *27.* ***Bibl.:*** *22.*

Malyarets Lyudmyla M. *– Doctor of Sciences (Economics), Professor, Head of the Department of Economic and Mathematical Modeling, Simon Kuznets Kharkiv National University of Economics (9a Nauky Ave., Kharkiv, 61166, Ukraine)*

E-mail: *malyarets@ukr.net*

ORCID: *https://orcid.org/0000-0002-1684-9805*

Researcher ID: *https://www.webofscience.com/wos/author/record/T-9858-2018*

Scopus Author ID: *https://www.scopus.com/authid/detail.uri?authorId=57189248374*

Voronin Anatolii V. *– Candidate of Sciences (Engineering), Associate Professor, Associate Professor of the Department of Economic and Mathematical Modeling, Simon Kuznets Kharkiv National University of Economics (9a Nauky Ave., Kharkiv, 61166, Ukraine)*

E-mail: *voronin61@ ukr.net*

ORCID: *https://orcid.org/0000-0003-1662-6035*

Scopus Author ID: *https://www.scopus.com/authid/detail.uri?authorId=58677148800*

Lebedeva Irina L. *– Candidate of Sciences (Physics and Mathematics), Associate Professor, Associate Professor of the Department of Economic and Mathematical Modeling, Simon Kuznets Kharkiv National University of Economics (9a Nauky Ave., Kharkiv, 61166, Ukraine)*

E-mail: *irina.lebedeva@hneu.net*

ORCID: *https://orcid.org/0000-0002-0381-649X*

Scopus Author ID: *https://www.scopus.com/authid/detail.uri?authorId=57196850420*

Lebedev Stepan S. *– Senior Lecturer of the Department of Economic and Mathematical Modeling, Simon Kuznets Kharkiv National University of Economics (9a Nauky Ave., Kharkiv, 61166, Ukraine)*

E-mail: *stepan.lebedev@hneu.net*

ORCID: *https://orcid.org/0000-0001-9617-7481*

Scopus Author ID: *https://www.scopus.com/authid/detail.uri?authorId=58677849900*

* The Fundamental, State Budget-Funded Research Work «Ensuring the Innovative Development of Export-Import Activities of Economic Entities in the Public Sector of the Economy under the Martial Law and in the Post-War Conditions». State Registration No. 0123U102012.

УДК 336.443:338.242.4.025.2
JEL Classification: C52; E52; F31

Маляреćь Л. М., Воронін А. В., Лєбєдєва І. Л., Лебедєв С. С. Математичне моделювання динаміки обмінного курсу як основа для державної регуляторної політики

Успішність державного регулювання валютного ринку, від якого залежить стабілізація економіки країни, підтримка конкурентоспроможності експорту, контроль інфляції тощо, визначається тим, яку модель покладено в основу прийняття управлінських рішень. У роботі запропоновано два підходи до розв'язання задачі оптимального регулювання курсу валют. Один із підходів базується на використанні рівномірночастотного критерію якості управління, а інший – на застосуванні інтегрального квадратичного критерію якості управління процесом балансування між попитом та пропозицією на валютному ринку. Мета роботи полягала в побудові математичної моделі, яка б дозволяла визначати оптимальні параметри валютного ринку і умови, за яких відхилення реального валютного курсу від необхідного фіксованого курсу було б мінімальним. У межах запропонованої в роботі математичної моделі вважається, що всі процеси, які розглядаються, є неперервними у часі. Це дозволяє застосовувати для побудови моделі елементи інтегрального та диференціального числення. Отримана математична модель являє собою систему диференціальних рівнянь і містить два типи критеріїв якості, які на основі аналізу можливих коливань валютного курсу дозволяють вибрати способи його регулювання. Однак ці критерії мають різну спрямованість. Так, застосування рівномірночастотного критерію спрямовано на зниження коливності курсових відхилень від необхідного значення за найгіршого впливу сезонних коливань валютного попиту. Тоді як реалізація регулювання курсу за допомогою інтегрального квадратичного критерію якості керування дозволяє оптимізувати витрати на стабілізацію курсу на тривалі часові інтервали. Отримані результати дозволяють використовувати запропоновану математичну модель для обґрунтування вибору шляхів оптимізації регуляторної політики держави на валютному ринку.

Ключові слова: *рівномірночастотний критерій, валютний курс, валютний ринок, математична модель.*

Формул: *27.* ***Бібл.:*** *22.*

Маляреćь Людмила Михайлівна *– доктор економічних наук, професор, завідувач кафедри економіко-математичного моделювання, Харківський національний економічний університет імені Семена Кузнеця (просп. Науки, 9а, Харків, 61166, Україна)*

E-mail: *malyarets@ukr.net*

ORCID: *https://orcid.org/0000-0002-1684-9805*

Researcher ID: *https://www.webofscience.com/wos/author/record/T-9858-2018*

Scopus Author ID: *https://www.scopus.com/authid/detail.uri?authorId=57189248374*

Воронін Анатолій Віталійович *– кандидат технічних наук, доцент, доцент кафедри економіко-математичного моделювання, Харківський національний економічний університет імені Семена Кузнеця (просп. Науки, 9а, Харків, 61166, Україна)*

E-mail: *voronin61@ ukr.net*

ORCID: *https://orcid.org/0000-0003-1662-6035*

Scopus Author ID: *https://www.scopus.com/authid/detail.uri?authorId=58677148800*

Лєбєдєва Ірина Леонідівна *– кандидат фізико-математичних наук, доцент, доцент кафедри економіко-математичного моделювання, Харківський національний економічний університет імені Семена Кузнеця (просп. Науки, 9а, Харків, 61166, Україна)*

E-mail: *irina.lebedeva@hneu.net*

ORCID: *https://orcid.org/0000-0002-0381-649X*

Scopus Author ID: *https://www.scopus.com/authid/detail.uri?authorId=57196850420*

Лебедєв Степан Сергович *– старший викладач кафедри економіко-математичного моделювання, Харківський національний економічний університет імені Семена Кузнеця (просп. Науки, 9а, Харків, 61166, Україна)*

E-mail: *stepan.lebedev@hneu.net*

ORCID: *https://orcid.org/0000-0001-9617-7481*

Scopus Author ID: *https://www.scopus.com/authid/detail.uri?authorId=58677849900*

Introduction. Regulating the exchange rate is an important and rather complex problem faced by both developed countries and those with transitional economies. Managing the exchange rate is part of the State's economic policy. As known from global practice, currency operations are not always conducted for the purpose of financial calculations. Often, activities in the foreign exchange market imply obtaining speculative profits. In turn, this has a significant impact on the process of exchange rate formation.

At present, most countries tend to utilize a floating exchange rate regime. However, in this case, sharp fluctuations in the rate may occur, leading to the necessity of stabilization, which is undertaken by the central bank. The objective of such managerial influence is to stabilize the economy, maintain the competitiveness of exports, control inflation, and protect against external shocks. A stable exchange rate makes the country more attractive to foreign investors, reducing the risks of losses due to currency fluctuations.

It should be emphasized that in the process of forming a currency stabilization policy, one faces the challenge of balancing stability and flexibility. Excessive intervention may lead to crises, while complete non-intervention may result in instability and a loss of control over the economy. Currently, there is no clear understanding of the theory of currency regulation, including the forms of currency policy that may be most applicable in a specific economic situation. Since the vectors of currency regulation affect all areas of economic activity, the correct choice of the exchange rate regime, various forms of currency policy, and methods of currency control that would align most closely with economic reforms is one of the priority tasks of economic theory and practice. In this context, there is a need to develop mathematical models that would serve as the basis for selecting optimal parameters of the equilibrium exchange rate when forming the State's currency policy. While constructing such models, it is essential to consider not only general development trends but also the cyclicality of economic processes and potential deviations from the overall tendency, also caused by force majeure circumstances. The creation of such mathematical models allows for identifying priority types of economic activity and the significance of their impact on the economic development of the country [1–3 et al.]. At this, the analysis of the dynamics of the monetary cycle plays a special role [4–7 et al.]. Although mathematical models are widely used in various economic research, they are primarily models developed for forecasting exchange rate dynamics [8–13 et al.]. The construction of such models mainly employs the method of correlation-regression analysis of time series, genetic algorithm, as well as methods of fractal geometry. Also, significant attention is given to the analysis of the dynamics of general economic equilibrium and the corresponding real exchange rate based on the equilibrium states of the external and internal balances of the country.

The mathematical models for regulating exchange rates, which are the focus of this study, generally represent a system of equations and algorithms. Analyzing potential fluctuations in the exchange rate allows for the selection of optimal methods for its management, as well as assessing the effectiveness of the central bank's or government's policies. When constructing models of this type, the main attention is given to the fundamental factor models that determine the structure of the balance of payments and, consequently, the exchange rate. It is assumed that the equilibrium exchange rate is determined by supply and demand in the financial asset market, taking into account the competition of open economies [14–18 et al.].

The aim of the study is to develop a mathematical model that would allow for the selection of optimal parameters for the State regulation in the currency market while ensuring conditions under which the deviation of the actual exchange rate from the required fixed rate is minimal.

Results and discussion. The studies [19; 20] present a mathematical model of the mechanism of functioning of the currency market, based on the interaction of non-speculative operators (commercial agents), speculative players, and the State. In this model, the excess demand created by non-speculative participants depends on the current exchange rate and seasonal factors that have a periodic structure of the following:

$$D_1(t) = a_0 - a_1 C(t) + B_0 \cos(\omega_0 t) \qquad (1)$$

with such constraints on the constant parameters:

$$a_0 > 0, \quad a_1 > 0, \quad 0 < B_0 < a_0,$$

where $C(t)$ is the current exchange rate of the local currency; B_0, ω_0 are the amplitude and frequency of fluctuations in the seasonal component of demand, accordingly; a_0 is the constant component of demand; a_1 is the elasticity of demand with respect to the exchange rate.

Let us consider the behavior of speculative agents whose demand and supply in the currency market are determined by the assumption that the exchange rate is unstable. Their excess demand for currency exchange can be determined by the formula:

$$D_2(t) = m\big(E(t) - C(t)\big), \quad m > 0, \qquad (2)$$

Where $E(t)$ is the expected exchange rate, m is the adjustment parameter.

To determine the trajectory of the exchange rate, we propose the following hypothesis about the formation of expectations. Let us assume that the structure of the expected exchange rate is determined by the following second-order differential equation:

$$E(t) = C(t) - b_1 \frac{dC(t)}{dt} - b_2 \frac{d^2 C(t)}{dt^2}, \quad b_1 > 0, \quad b_2 > 0.$$

This means that speculative players base their expectations on the current exchange rate level and on the regressive direction of its change (the first derivative), taking into account the acceleration of this change (the second derivative). Let us assume that a government monetary policy is being implemented, the goal of which is to stabilize the exchange rate at a constant level $C^* = a_0$. In this case, the excess demand of the State is described by the equation:

$$D_3(t) = f_1\big(C^* - C(t)\big) - f_2 \frac{dC}{dt}, \qquad (3)$$

where f_1, f_2 are the State regulators, i. e., parameters determined by the State policy.

The market balance is defined by the formula:

$$D_1(t) + D_2(t) + D_3(t) = 0. \qquad (4)$$

After substituting expressions (1)-(3) into the balance equation (4), we obtain a second-order differential equation with respect to the variable $x(t) = C(t) - C^*$, which describes the deviation of the current exchange rate from the required value. After transformation, this equation takes the form:

$$\frac{d^2 x}{dt^2} + (h + k_2)\frac{dx}{dt} + (g + k_1)x = B\cos(\omega_0 t), \qquad (5)$$

where

$$g = \frac{a_1}{mb_2}, \quad h = \frac{b_1}{b_2}, \quad k_1 = \frac{f_1}{mb_2}, \quad k_2 = \frac{f_2}{mb_2}, \quad B = \frac{B_0}{mb_2}.$$

Using the terminology of automatic control theory, we rewrite equation (5) in operator form as a system:

$$\begin{cases} \left(p^2 + hp + g\right) \cdot x(p) = U(p) + W(p), \\ U(p) = -\left(k_2 p + k_1\right) \cdot x(p), \end{cases} \qquad (6)$$

Where $p = \dfrac{d}{dt}$ is the differentiation operator; $x(p)$ is the input characteristic of the object; $U(p)$ is the control, chosen in the form of negative feedback based on the output and its derivative; $W(p)$ is an external disturbing periodic influence.

Let us introduce the transfer function of the output in relation to the external input influence $H(p)=\dfrac{x(p)}{W(p)}$ as: $H(p)=\dfrac{1}{p^2+(h+k_2)p+g+k_1}$, and the transfer function of control in relation to the disturbance $G(p)=\dfrac{U(p)}{W(p)}$ as: $G(p)=-\dfrac{k_2p+k_1}{p^2+(h+k_2)p+g+k_1}$. It is evident that $H(p)$ and $G(p)$ are stable realizable operators, as the degrees of the polynomial numerators are strictly less than the degrees of the polynomial denominators and the roots of the denominators have negative real parts.

The external disturbance $W(t)=B\cos(\omega_0 t)$ may have an arbitrary frequency ω_0. Then the guaranteed estimate of the amplitude of the steady-state deviation of the exchange rate is equal to:

$$\delta=\sup_{\omega}\left|H(p)\right|_{p=i\omega},\quad i^2=-1.$$

It is natural to call this quantity [21] a uniformly frequented reaction indicator of system (6). In classical control theory, the term «volatility index» was used. In function theory, this same quantity is defined as the norm of the function $H(p)$ of a complex variable p in Hardy space, denoted by the standard notation: $\|H\|_\infty=\sup_{\omega}|H(i\omega)|$. The objective of optimal control consists of synthesizing a linear regulator in the form of system (2), which stabilizes control and linearizes the quality functional (7).

$$J=\inf_{k_1,k_2}\sup_{\omega}\|F(i\omega)\|_\infty^2, \tag{7}$$

where $F(p)=\dfrac{Z(p)}{W(p)}$ is the transfer function of the generalized output of system (6) in response to disturbances. Typically, a linear combination of the standard output $x(p)$ and control $U(p)$ is used as of the generalized output $Z(p)$:

$$Z(p)=r(qx(p)+U(p)), \tag{8}$$

where r, q are constant parameters.

The parameter r is an indicator of management costs, while the product rq is interpreted as a characteristic of control accuracy, i. e., the quantity q reflects a quantitative measure of the compromise between achievable accuracy and regulation costs. Considering formulas (7) and (8), we obtain an expression for the quality functional:

$$J=\inf_{k_1,k_2}\sup_{\omega}r^2\left(q^2\left|H(i\omega)\right|^2+\left|G(i\omega)\right|^2\right), \tag{9}$$

where $\left|H(i\omega)\right|^2=H(i\omega)H(-i\omega)$,

$\left|G(i\omega)\right|^2=G(i\omega)G(-i\omega)$.

After substituting the explicit expressions for the transfer functions H and G into the functional (9), we obtain:

$$J=\inf_{k_1,k_2}\sup_{\omega}\frac{r^2(q^2+k_1^2+k_2^2\omega^2)}{\omega^4+((h+k_2)^2-2(g+k_1))\omega^2+(g+k_1)^2}. \tag{10}$$

The quality criterion (10), as a result of maximizing over the frequency ωtakes the form:

$$J^*(k_1,k_2)=\frac{r^2k_2^4}{r\sqrt{k_1^2+g^2+(g+k_1)k_2^2}-2\left(k_1+g^2+(k_1+g)k_2^2\right)+k_2^2(h+k_2)^2}. \tag{11}$$

From the necessary conditions for the existence of an extremum, we obtain a system of algebraic equations concerning k_1 and k_2, the solution of which is the solution to the optimization problem (11).

$$\begin{cases}k_1^3+\left(2g-\dfrac{h^2}{2}\right)k_1^2-q^2k_1-hk_1^2k_2+\left(gk_1-\dfrac{q^2}{2}\right)k_2^2=0,\\ A_5k_2^5+A_4k_2^4+A_3k_2^3+A_2k_2^2+A_1k_2+A_0=0,\end{cases} \tag{12}$$

where

$$A_5=h^2(n_1^2-n_2),\quad A_4=-2h((2n_1-h^2)(n_1^2-n_2)+n_2h^2),$$

$$A_3=(n_1^2-n_2)(2n_1-h^2)^2-4(n_1^2-n_2)+5n_2h^2(2n_1-h^2),$$

$$A_2=4n_2h(3(n_1^2-n_2)+(2n_1-h^2)^2),$$

$$A_1=n_2(2n_1-h^2)^3-4n_2(2n_1-h^2)(n_1^2-n_2)-8n_2^2h^2,$$

$$A_0=4n_2^2h(2n_1-h^2),\quad n_1=g+k_1,\quad n_2=q^2+k_1.$$

The system of algebraic equations (12) evidently cannot be explicitly solved for the unknowns k_1^* and k_2^*. In this case, it becomes necessary to apply numerical methods to obtain approximate values for the sought quantities.

Let us examine in detail the situation where government regulation policies slightly consider the rate of change of the exchange rate difference, that is, the coefficient $k_2=\mu$ is a small quantity, and the quantities containing k_2 raised to higher than one can be neglected. In this case, functional (10) will take the form:

$$J=\inf_{k_1}\sup_{\omega}\frac{r^2(q^2+k_1^2)}{\omega^4+h(h_0^2-2(g+k_1))\omega^2+(g+k_1)^2}, \tag{13}$$

where $h_0^2=(h+\mu)^2=h^2+2\mu$.

Then for the case when the frequency is maximal, the criterion (13) is written as follows:

$$J^*(k_1)=\inf_{k_1}\frac{r^2(q^2+k_1^2)}{h_0^2\left(k_1+g-\dfrac{h_0^2}{4}\right)}. \tag{14}$$

From the necessary condition of the extremum $\frac{dJ^*(k_1)}{dk_1}=0$ we obtain the equation for determining the optimal value of k_1*:

$$k_1^2+\left(2g-\frac{h_0^2}{2}\right)k_1-q^2=0. \quad (15)$$

It is evident that the roots of the quadratic equation (15) have different signs. Hence, the solution is the positive root, namely

$$k_1^*=\frac{h_0^2}{4}-g+\sqrt{\left(\frac{h_0^2}{4}-g\right)^2+q^2}. \quad (16)$$

Substituting the optimal value of k_1* into the original expression for the quality criterion allows us to find its minimum value $J^*(k_1^*)=\gamma^2$:

$$\gamma^2=\frac{2r^2k_1^*}{h_0^2}, \text{ or } \gamma^2=\frac{2r^2}{h_0^2}\left(\frac{h_0^2}{4}-g+\sqrt{\left(\frac{h_0^2}{4}-g\right)^2+q^2}\right), \quad (17)$$

The quantity $\gamma=\frac{r}{h_0}\sqrt{2k_1^*}$ represents the maximum deviation from the required level of the generalized output of the original dynamic system (6). In the case where the parameter r is small, i. e., the State implements a «cheap» management conception, the required accuracy of stabilization can be absolute, as the relation $\gamma\to 0$ [22] holds.

Using expression (16), which explicitly shows k_1* we will calculate the maximum value of the amplitude of deviations of the exchange rate from the required level, i. e., $|X(t)|\le\delta$:

$$\delta=\frac{1}{h_0\sqrt[4]{\left(\frac{h_0^2}{4}-g\right)^2+q^2}}. \quad (18)$$

In the absence of exchange rate regulation by the State, i. e., when $k_1 = k_2 = 0$, respectively, $q = 0$. For this case, we obtain the value of the amplitude of the exchange rate:

$$\delta_0=\frac{1}{h_0\sqrt{g-\frac{h_0^2}{4}}}, \quad g>\frac{h^2}{4}.$$

Let us establish a relationship $\xi=\frac{\delta_0}{\delta}$:

$$\xi=\sqrt[4]{1+\frac{q^2}{\left(\frac{h_0^2}{4}-g\right)^2}}.$$

It is evident that $\xi > 1$, therefore, the State policy of stabilizing the exchange rate, proportional to the deviation from the required level $C^*=\frac{a_0}{a_1}$, allows for a reduction in the maximum possible amplitude of exchange rate fluctuations by ξ times.

Let us express the quantity δ in terms of the model's initial data (5):

$$\delta=\frac{b_2^2}{b_1\sqrt[4]{\left(\frac{b_1^2}{4}-\frac{a_1b_2}{m}\right)^2+q^2b_2^4}}. \quad (19)$$

Using (19), it is not difficult to determine the lower and upper bounds for the current exchange rate:

$$\frac{a_0}{a_1}-\delta<C(t)<\frac{a_0}{a_1}+\delta.$$

This can serve as a methodological basis for substantiating the parameters of the currency corridor.

The optimal parameter of the State policy

$$f_1^*=\frac{mb_1^2}{4b_2}-a_1+\sqrt{\left(\frac{mb_1^2}{4b_2}-a_1\right)^2+m^2b_2^2q^2},$$

while $f_2^*=\gamma$ defines the amount of foreign currency required by the government to maintain the national monetary unit at a specified level.

Let us consider another approach to solving the problem at hand. The functional (9) can be presented in a slightly different form:

$$J=\inf_{k_1,k_2}\frac{1}{2\pi}\int_{-\infty}^{+\infty}r^2(q^2|H(i\omega)|^2+|G(i\omega)|^2)d\omega. \quad (20)$$

This integral quadratic quality criterion corresponds to the norm $\|H\|_2=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|H(i\omega)|^2d\omega$, which in turn characterizes the quadratic deviation of the exchange rate from the required value. Then (10) is transformed into the form:

$$J_2=\inf_{k_1,k_2}\sup_{\omega}\frac{r^2}{2\pi}\int_{-\infty}^{+\infty}\frac{(q^2+k_1^2+k_2^2\omega^2)d\omega}{\omega^4+((h+k_2)^2-2(g+k_1))\omega^2+(g+k_1)^2}. \quad (21)$$

Integrating with respect to frequency ω, we obtain in this case such an expression for the quality criterion explicitly as a function of k_1 and k_2:

$$J_2=\inf_{k_1,k_2}\frac{r^2}{2}\left(\frac{q^2+k_1^2}{(h+k_2)(g+k_1)}+\frac{k_2^2}{h+k_2}\right). \quad (22)$$

From one of the necessary conditions for the extremum $\frac{\partial J_2}{\partial k_1}=0$ we derive the equation for finding the optimal value of k_1:

$$k_1^2+2gk_1-q^2=0. \quad (23)$$

Thus, we find that

$$k_1^* = \sqrt{g^2 + q^2} - g. \quad (24)$$

From another necessary condition for the extremum $\frac{\partial J_2}{\partial k_2} = 0$ we derive the equation for finding the optimal value of k_2:

$$k_2^2 + 2hk_2 - 2k_1 = 0. \quad (25)$$

Considering that in formula (24) the quantity k_1* is presented explicitly, we obtain the following expression for k_2* :

$$k_2^* = \sqrt{h^2 + 2(\sqrt{g^2 + q^2} - g)} - h. \quad (26)$$

By substituting relationships (24) and (26) into expression (22), we get the optimal value of the quality criterion:

$$J_2^* = r^2 k_2^* = r^2 \sqrt{h^2 + 2(\sqrt{g^2 + q^2} - g)} - h. \quad (27)$$

The formula (27) reflects the quantitative measure of the minimum square deviation of the exchange rate from its required value.

Conclusions. The main factors discussed in this study that affect the currency market facilitate the identification of processes that determine the formation of the exchange rate. The two approaches proposed in the article for optimal exchange rate regulation, namely, using uniform frequency and integral quadratic quality criteria, enable the determination of optimal parameters for regulatory policy. The use of one of the two proposed quality criteria in the mathematical model for regulating the formation of the exchange rate has different influences on its dynamics. Thus, the application of the uniform frequency criterion aims to reduce the volatility of exchange rate deviations from the desired value during the worst impact of seasonal fluctuations in currency demand. In contrast, implementing exchange rate regulation through the integral quadratic quality management criterion allows for the optimization of costs for stabilizing the exchange rate over long periods. This allows for effective management of the balancing process between supply and demand in the currency market.

LITERATURE

1. Bandura O. V. Optimization of macroeconomic policy and stabilization of cyclical economic dynamics. *Economy and Forecasting*. 2021. No. 4. P. 102–124. DOI: 10.15407/eip2021.04.102

2. Kosarchyn M. Global financial cycle: Impact on Ukraine. *Development Management*. 2023. Vol. 22 (4). C. 16–24.
DOI: 10.57111/devt/4.2023.16

3. Rayevnyeva O., Stryzhychenko K., Shtal T., Brovko O., Koinash M. Analysis of fluctuations in the national economy: Models and development scenarios. *Economics of Development*. 2025. Vol. 24 (1). P. 8–25. DOI: 10.57111/econ/1.2025.08

4. Dornbusch R. Exchange rate rules and macroeconomic stability. In: Williamson J. (ed.) *Exchange Rate Rules*. London : Palgrave Macmillan, 1981. P. 55–67.
DOI: 10.1007/978-1-349-05166-3_3

5. Challoumis C. Analysis of the theory of cycle of money. *Acta Univ. Bohem. Merid.* 2020. Vol. 23 (2). C. 13–29.
DOI: 10.2478/acta-2020-0004

6. Morina F., Hysa E., Ergün U., Panait M., Voica M. C. The effect of exchange rate volatility on economic growth: Case of the CEE countries. *Journal of Risk and Financial Management*. 2020. Vol. 13 (8). Article 177. DOI: 10.3390/jrfm13080177

7. Challoumis C. Integrating money cycle dynamics and economocracy for optimal resource allocation and economic stability. *Journal of Risk and Financial Management*. 2024. Vol. 17 (9). Article 422. DOI: 10.3390/jrfm17090422

8. Колодізєв О. М. Прогнозування валютних курсів: макро- та мікроекономічні аспекти : монографія. Харків : ВД «ІНЖЕК», 2008. 352 с.

9. Сергієнко О. А., Татар М. С. Моделі прогнозування валютних курсів у системі управління конкурентоспроможністю підприємства. *Проблеми економіки*. 2013. № 2. С. 268–278.

10. Кривда О. В., Сидоренко Ю. В., Романова Д. П. Прогнозування динаміки економічних процесів за допомогою методів фрактальної геометрії. *Економічний бюлетень Національного технічного університету України "Київський політехнічний інститут"*. 2017. № 14. С. 483–490.

11. Amat C., Michalski T., Stoltz G. Fundamentals and exchange rate forecastability with simple machine learning methods. *Journal of International Money and Finance*. 2018. Vol. 88. P. 1–24. DOI: 10.1016/j.jimonfin.2018.06.003

12. Zhang Y., Hamori S. The predictability of the exchange rate when combining machine learning and fundamental models. *Journal of Risk and Financial Management*. 2020. Vol. 13 (3). Article 48. DOI: 10.3390/jrfm13030048

13. Мельник О., Новоселецький О. Економіко-математичні інструменти для прогнозування валютного курсу. *Науковий погляд: Економіка та управління*. 2022. Т. 78 (2). С. 173–178.

14. Taylor L. Exchange rate indeterminacy in portfolio balance, Mundell–Fleming and uncovered interest rate parity models. *Cambridge Journal of Economics*. 2004. Vol. 28 (2). P. 205–227. DOI: 10.1093/cje/28.2.205

15. Bianchi S., Pantanella A., Pianese A. Modeling and simulation of currency exchange rates using multifractional process with random exponent. *International Journal of Modeling and Optimization*. 2012. Vol. 2 (3). P. 309–314.

16. Li M., Qin F., Zhang Z. Short-term capital flows, exchange rate expectation and currency internationalization: Evidence from China. *Journal of Risk and Financial Management*. 2021. Vol. 14 (5). Article 223. DOI: 10.3390/jrfm14050223

17. Kuzmin A. Mathematical exchange rates modeling: Equilibrium and nonequilibrium dynamics. *Mathematics*. 2022. Vol. 10 (24). Article 4672. DOI: 10.3390/math10244672

18. Tariffi L. A general equilibrium model with real exchange rates. *Economies*. 2025. Vol. 13 (5). Article 122.
DOI: 10.3390/economies13050122

19. Gandolfo G. Economic Dynamics. 3rd ed. Berlin : Springer, 1996.

20. Лисенко Ю. Г., Петренко В. Л., Тимохін В. Н., Філіпов А. В. Економічна динаміка : підручник. Донецьк : ДонНУ, 2010.

21. Francis B. A. A Course in H∞ Control Theory. *Lecture Notes in Control and Information Sciences*. Vol. 88. Berlin–Heidelberg–New York : Springer-Verlag, 1977.

22. Khargonekar P. P., Rotea M. A. Mixed H_2|H∞ control: a convex optimization approach. *IEEE Transactions on Automatic Control*. 1991. Vol. 36 (7). P. 824–837.

REFERENCES

Amat, C., Michalski, T., and Stoltz, G. "Fundamentals and exchange rate forecastability with simple machine learning methods". *Journal of International Money and Finance*, vol. 88 (2018): 1-24. DOI: 10.1016/j.jimonfin.2018.06.003

Bandura, O. V. "Optimization of macroeconomic policy and stabilization of cyclical economic dynamics". *Economy and Forecasting*, no. 4 (2021): 102-124. DOI: 10.15407/eip2021.04.102

Bianchi, S., Pantanella, A., and Pianese, A. "Modeling and simulation of currency exchange rates using multifractional process with random exponent". *International Journal of Modeling and Optimization*, vol. 2 (3) (2012): 309-314.

Challoumis, C. "Analysis of the theory of cycle of money". *Acta Univ. Bohem. Merid*, vol. 23 (2) (2020): 13-29. DOI: 10.2478/acta-2020-0004

Challoumis, C. "Integrating money cycle dynamics and economocracy for optimal resource allocation and economic stability". *Journal of Risk and Financial Management*, art. 422, vol. 17 (9) (2024). DOI: 10.3390/jrfm17090422

Dornbusch, R. "Exchange rate rules and macroeconomic stability". In *Exchange Rate Rules*, 55-67. London: Palgrave Macmillan, 1981. DOI: 10.1007/978-1-349-05166-3_3

Francis, B. A. "A Course in H? Control Theory". In *Lecture Notes in Control and Information Sciences*, vol. 88. Berlin; Heidelberg; New York: Springer-Verlag, 1977.

Gandolfo, G. *Economic Dynamics*. Berlin: Springer, 1996.

Khargonekar, P. P., and Rotea, M. A. "Mixed H?|H? control: a convex optimization approach". *IEEE Transactions on Automatic Control*, vol. 36 (7) (1991): 824-837.

Kolodiziev, O. M. *Prohnozuvannia valiutnykh kursiv: makro- ta mikroekonomichni aspekty* [Forecasting Exchange Rates: Macroeconomic and Microeconomic Aspects]. Kharkiv: VD «INZhEK», 2008.

Kosarchyn, M. "Global financial cycle: Impact on Ukraine". *Development Management*, vol. 22 (4) (2023): 16-24. DOI: 10.57111/devt/4.2023.16

Kryvda, O. V., Sydorenko, Yu. V., and Romanova, D. P. "Prohnozuvannia dynamiky ekonomichnykh protsesiv za dopomohoiu metodiv fraktalnoi heometrii" [Forecasting the Dynamics of Economic Processes Using Fractal Geometry Methods]. *Ekonomichnyi biuleten Natsionalnoho tekhnichnoho universytetu Ukrainy "Kyivskyi politekhnichnyi instytut"*, no. 14 (2017): 483-490.

Kuzmin, A. "Mathematical exchange rates modeling: Equilibrium and nonequilibrium dynamics". *Mathematics*, art. 4672, vol. 10 (24) (2022). DOI: 10.3390/math10244672

Li, M., Qin, F., and Zhang, Z. "Short-term capital flows, exchange rate expectation and currency internationalization: Evidence from China". *Journal of Risk and Financial Management*, art. 223, vol. 14 (5) (2021). DOI: 10.3390/jrfm14050223

Lysenko, Yu. H. et al. *Ekonomichna dynamika* [Economic Dynamics]. Donetsk: DonNU, 2010.

Melnyk, O., and Novoseletskyi, O. "Ekonomiko-matematychni instrumenty dlia prohnozuvannia valiutnoho kursu" [Economic and Mathematical Tools for Forecasting the Exchange Rate]. *Naukovyi pohliad: Ekonomika ta upravlinnia*, vol. 78 (2) (2022): 173-178.

Morina, F. et al. "The effect of exchange rate volatility on economic growth: Case of the CEE countries". *Journal of Risk and Financial Management*, art. 177, vol. 13 (8) (2020). DOI: 10.3390/jrfm13080177

Rayevnyeva, O. et al. "Analysis of fluctuations in the national economy: Models and development scenarios". *Economics of Development*, vol. 24 (1) (2025): 8-25. DOI: 10.57111/econ/1.2025.08

Serhiienko, O. A., and Tatar, M. S. "Modeli prohnozuvannia valiutnykh kursiv u systemi upravlinnia konkurentospromozhnistiu pidpryiemstva" [Models for Forecasting Exchange Rates in the Enterprise Competitiveness Management System]. *Problemy ekonomiky*, no. 2 (2013): 268-278.

Tariffi, L. "A general equilibrium model with real exchange rates". *Economies*, art. 122, vol. 13 (5) (2025). DOI: 10.3390/economies13050122

Taylor, L. "Exchange rate indeterminacy in portfolio balance, Mundell-Fleming and uncovered interest rate parity models". *Cambridge Journal of Economics*, vol. 28 (2) (2004): 205-227. DOI: 10.1093/cje/28.2.205

Zhang, Y., and Hamori, S. "The predictability of the exchange rate when combining machine learning and fundamental models". *Journal of Risk and Financial Management*, art. 48, vol. 13 (3) (2020). DOI: 10.3390/jrfm13030048

Стаття надійшла до редакції 02.05.2025 р.
Статтю прийнято до публікації 19.06.2025 р.