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NONLINEAR REGULATORS OF INNOVATION PROCESSES

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This study considers the methodology of synergistic management of innovation processes. As a basic mathematical model of innovation dynamics as an economic process, an ordinary first-order differential equation with quadratic nonlinearity is assigned. The specificity of this differential equation is the presence of two special solutions - equilibrium positions, which determines the fundamental difference of this mathematical model from linear dynamics models. The very fact of the existence of two equilibrium positions inherent in the innovative system makes it relevant to formulate the problem of synthesizing nonlinear regulators in order to ensure that the system achieves the desired dynamic trajectories (attractors) through the use of the theory of synergistic control of significantly nonlinear objects. One of the main results of this study is the analysis of the influence of a nonlinear (quadratic) controller on the properties of a dynamic model, which is presented in the form of a traditional logistic-type differential equation. The conditions for obtaining the characteristic values of the controller parameters for a system with two equilibrium positions are determined. It is found that a greater value of the characteristic indicator of the equilibrium position corresponds to a stable equilibrium position of the innovative system, and a smaller value corresponds to an unstable one. This fact confirms that the application of the conception of synergistic control does not change the basic properties of dynamic processes "with saturation", that is, the diffusion of innovations is a process that has a limit from above. It is proved that when both equilibrium positions are close in value of the characteristic indicator, a saddle-nodal bifurcation occurs, and an analysis of the mechanism of loss of stability occurring in the system is also carried out. It is shown that this can lead to undesirable effects, such as a catastrophic loss of stability of the "fold" type. Therefore, when implementing synergistic control, which is nonlinear in nature, it is necessary to prevent the emergence of negative trends in innovative development by choosing such regulator parameters that allow maintaining the stability of innovation diffusion processes near the upper equilibrium position.

Keywords: *diffusion of innovations; nonlinear dynamics; synergistic management; self-organization; logistic function; attractor; nonlinear regulator; feedback.*

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Воронін А. В., Лебедєв С. С. Нелінійні регулятори інноваційних процесів

У цьому дослідженні розглянуто методологію синергетичного керування інноваційними процесами. Як базову математичну модель інноваційної динаміки як економічного процесу представлено звичайне диференціальне рівняння першого порядку з квадратичною нелінійністю. Специфікою цього диференціального рівняння є наявність двох особливих розв'язків – положень рівноваги, що визначає принципову відмінність цієї математичної моделі від моделей лінійної динаміки. Сам факт існування двох положень рівноваги, притаманних інноваційній системі, робить актуальною постановку задачі синтезу нелінійних регуляторів з метою забезпечити досягнення системою бажаних динамічних траєкторій (атракторів) завдяки використанню теорії синергетичного керування суттєво нелінійними об'єктами. Одним із основних результатів цього дослідження є аналіз впливу нелінійного (квадратичного) регулятора на властивості динамічної моделі, яка представлена у вигляді традиційного диференціального рівняння логістичного типу. Визначено умови для отримання характеристичних значень параметрів регулятора для системи з двома положеннями рівноваги. Встановлено, що більше за значенням характеристичного показника положення рівноваги інноваційної системи є стійким, а менше за значенням – нестійким. Цей факт підтверджує, що застосування концепції синергетичного керування не змінює базових властивостей динамічних процесів «з насиченням», тобто дифузія інновацій є процесом, що має обмеження зверху. Доведено, що коли обидва положення рівноваги є близькими за значенням характеристичного показника, виникає сідло-вузлова біфуркація. Також проведено аналіз механізму втрати стійкості, що відбувається в системі. Показано, що це може призвести до небажаних ефектів, зокрема катастрофічної втрати стійкості типу «складка». Отже, під час реалізації синергетичного керування, яке є нелінійним за своєю природою, треба запобігати появі негативних тенденцій інноваційного розвитку завдяки вибору таких параметрів регулятора, які дозволяють зберігати стійкість процесів дифузії інновацій поблизу верхнього положення рівноваги.

Ключові слова: *дифузія інновацій; нелінійна динаміка; синергетичне управління; самоорганізація; логістична функція; атрактор; нелінійний регулятор; зворотний зв'язок.*

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Introduction. The current state of economic science is focused on finding innovative ways to sustainably develop global economic processes. This modern paradigm of societal development assumes that meeting the needs of the present generation must occur without compromising the opportunities of future generations [1]. It has a systemic synergistic nature, since its implementation is determined by the synthesis of economic, environmental and social structural components, the evolution of which must be coordinated at the global, national and regional levels. It is known that the manifestation of joint coordinated actions can be the emergence of a synergistic effect in a complex system, when the characteristics of the state of this system quantitatively and qualitatively exceed the simple sum (superposition) of the corresponding characteristics of the individual elements of this system [2–4]. It is the scientifically based integration policy of states at the global level, as well as the adherence of industries and individual enterprises to decisions verified at the state level, that contribute to the emergence of a synergistic effect, which ensures increased effectiveness of national programs for achieving the Sustainable Development Goals [2–7], which were adopted by all 193 UN member states within the framework of the 2030 Agenda for Sustainable Development. In this regard, an important task of economic theory is to improve the understanding of the structure of these Goals, their main cause-and-effect relationships, as well as the influence of political decisions on the possible results of their implementation [8–10].

Accordingly, the question arises as to what analytical methods and mathematical models are appropriate to use to obtain a comprehensive understanding of the factors that determine the behavior of such systems. In addition, to construct these models, a rather complex mathematical apparatus is used, and carrying out transformations requires the use of a number of simplifications, i.e., not the general, but some specific particular cases are considered. Accordingly, such mathematical models are mainly focused on describing particular cases of the result of the interaction of economic processes occurring in the system. From this perspective, the synergistic effect is viewed as a catalyst that increases the efficiency of business processes at the level of individual enterprises or industries as a result of integration and the merging of disparate processes into a powerful, coherent system [11–13]. However, questions related to the dynamics of transformation of economic system states at the macro level remain largely open. One such question is the modeling of innovation diffusion processes.

Innovative development is considered the main tool for achieving the Sustainable Development Goals by 2030, formulated by the UN. Moreover, each of the areas of innovative development ensures the achievement of not only one specific of the 17 Goals, but also serves as a guarantee of the implementation of the concept as a whole. Without the creation of latest technologies, the introduction of the advanced business models, the widespread use of digital technologies, and the transition to new types of energy, it is impossible to solve such global problems as ensuring environmental protection, combating poverty and inequality in the context of limited planetary resources, or achieving success in the fight against climate change. The exceptional role of innovation implementation is also emphasized in the list of Sustainable Development Goals. Thus, Goal 9 is considered to be Build resilient infrastructure, promote inclusive and sustainable industrialization and innovation [14]. It is precisely the focus on sustainable industrialization, combined with innovative development, that contributes to the release of dynamic and competitive economic forces. These economic forces, in turn, play a key role in the implementation of the latest technologies, ensuring their effective use in all areas of our lives. Innovative development plays a decisive role in finding optimal solutions to economic, environmental, and social problems, as well as the rational use of resources and energy efficiency. In this regard, decision-making must be guided by a scientifically sound program for innovative development, both at the state level and at the level of individual industries or individual enterprises. Moreover, these programs must be coordinated.

The basic economic premise for constructing a model of an innovation system, which can serve as a basis for substantiating a high-quality forecast of possible paths of its development, is the assertion that innovative evolution is determined by the interaction of three main factors, which include:

- statistical regularities that determine the multiple equilibrium balance of the functioning of an economic system;
- cyclical dynamics, which is characterized by the presence of cycles of varying durations;
- sociogenetic as the relationship between heredity, variability and selection in technological and economic systems.

When constructing mathematical models that describe the interaction of factors that determine the state of an economic system, a linear approximation is typically used. This paradigm assumes that all relationships between economic indicators are described by linear equations and inequalities. This means, first of all, that the change in the internal factor is directly proportional to the change in each of the external factors separately. Secondly, technological standards, prices, unit costs, and other model coefficients remain constant throughout the entire period under consideration Third, the interaction of external factors is additive, meaning the overall change in an internal factor is simply the sum of the individual effects of each external factor. Linear models are successfully used to describe system behavior over short time periods, within which process parameters can be considered constant. An example of linear mathematical models can be econometric models, which are used to solve optimization problems of linear programming.

However, recently it has become increasingly clear that the currently dominant linear paradigm of economic systems analysis is not capable of adequately describing rapid changes, unpredictable leaps, and unexpected qualitative transformations that occur as a result of complex interactions between individual components of the modern market process and, accordingly, such a paradigm cannot be used to construct adequate mathematical models of these processes. In this context, synergetics—a modern integrated science that studies the self-organization processes of open systems and encompasses virtually all modern fields of natural science, engineering, and economics—is increasingly developing. This generalized interdisciplinary science is based on the laws of nonlinear dynamics and the thermodynamics of irreversible processes. Over the past decades, synergetics, from the theory of nonequilibrium thermodynamics of dissipative systems proposed by the Belgian physicist and chemist Ilya Prigogine, has been transformed into a general theory of development, which has had a great influence on ideological concepts in various branches of science [15]. Incidentally, it was for his work in the field of thermodynamics of irreversible processes that Prigogine received the Nobel Prize in Chemistry. The essence of this relatively new scientific field is that spontaneous self-organization processes arise in open systems exchanging energy, matter, and information with the external environment. This, in turn, initiates the emergence from chaos of certain stable, ordered structures with qualitatively new properties. Prigogine himself called his theory a theory describing the «creation of structure» (in contrast to classical thermodynamics, which is a theory of the destruction of structure).

It should be noted that systems in which a synergistic effect can occur have two characteristic features. First, the system is open, meaning that energy, matter, and information are exchanged between the system and the external environment. Secondly, the behavior of the system's components is coherent, that is, consistent with each other. The main characteristic of self-organization in synergetics is the emergence of a new quality in the system, new properties that its individual elements did not possess.

Despite the significant achievements of modern synergetics, the concept of management has not received due development and generalization in it and, accordingly, has not taken its rightful place, although in many ways it determines the very essence of self-organization processes. In accordance with the basic principles of economic synergetics, its distinctive feature is precisely spontaneous self-organization, and the true meaning of the cooperative processes that arise in this case lies in the internal causes of the unpredictable self-organization of systems of various natures [16–19]. It was precisely thanks to the discovery of the effect of the causal mode of self-organization that synergetics has achieved outstanding results in the study of cooperative phenomena in the economic environment. In this case, the principle of so-called «downward causality» is realized, according to which the whole (the system) imposes restrictions on the freedom of choice of development paths for its components. Accordingly, the system's elements can no longer act independently, but must obey certain rules inherent to the entire system [20]. An example of such self-organization is the «Cobra Effect» [21]. When the snake population in India became too large, the country's authorities instituted a bounty for every cobra killed to reduce the population. However, resourceful residents chose not to kill cobras in the wild, but instead began breeding them. Upon learning of this, the authorities canceled the bounty, and the disappointed people then released the cobras into the wild. As a result, the number of cobras in the wild increased sharply compared to the initial level, i.e., the effect was exactly the opposite of what the authorities had expected when making the corresponding management decision.

Another similar example is the Great Hanoi Rat Massacre [22]. To motivate Hanoi residents to exterminate rats, the authorities offered a cash reward. But to receive this reward, all that was needed was a rat's tail. To increase their income, Hanoi residents didn't kill the rats, but released the tailless animals to breed further, and even began setting up rat farms. As a result, the rat population in Hanoi increased.

In both examples, the reward decree plays the role of an informational influence, i.e., a signal received by an open system (the population). A fluctuation in external energy arose, and it was assumed that this would result in a linear model of system development. According to this model, the more rats killed, the greater the monetary reward. However, instead, the system experienced a bifurcation: rat farms emerged, and commercial rat farming began to develop. In the open system, order was established at a new structural level.

The above results are fully consistent with Goodhart's Law, which states that a selected indicator that has been converted into a performance target loses its effectiveness as an indicator of the goal it was intended to represent [23]. It should be noted that when analyzing synergetic systems, «ascending» causality is traditionally considered, and in this case, Goodhart's law plays an important role in determining the direction of development of the system under consideration.

However, to effectively apply the ideas of nonlinear dynamics to the management of economic systems, understanding the causal mechanism of self-organization is insufficient. A fundamentally new approach to understanding the nature of the interaction between external control influences and self-governing processes is required. And this approach consists of a transition from the unpredictable behavior of the economic system according to the algorithm of the dissipative structure to the directed movement of the target efficiency indicator along the desired attractors (a relatively stable state or the trajectory of the system's development), to which all other variables of the dynamic system are adjusted. This is a method of directed self-organization of synthesized systems. With this approach, the attractor defines the essence of the process of controlling a self-organizing system in accordance with the stated goal. From an informational perspective, this method reflects the process of information reception, i.e., the active selection and translation of external signals into the system's internal language, which triggers causal self-organization. In the examples of the «Cobra Effect» or «Rat Massacre,» local residents perceived government decrees regarding monetary rewards as an informational trigger. The reception of this information served as the impetus for the system to shift to a qualitatively new state – the creation of snake and rat farms. In the language of dynamic systems theory, this translates into a certain ordered state. It is precisely self-management and directed self-organization of nonlinear systems that are the properties, the consideration of which forms a new approach to organizational management, i.e., there is a tendency to move from traditional methods of economic cybernetics to modern ideas of synergetics. Thus, in a number of works [24–27] the principles of modern nonlinear control theory have been developed, the scientific basis of which is the application of the principles of self-organization to the modern theory of automatic control.

The aim of this article is to identify the fundamental features of nonlinear control theory as applied to the problem of system synthesis. The object of the study is an economic system in which the diffusion of innovations occurs. Diffusion of innovations is understood as the process by which a specific innovation is transmitted through communication channels between members of a social system [28]. Innovations can be ideas, technologies, products, etc. that are new to society. Despite the different nature of innovative products and the differences between social groups in which innovations are spreading, the dynamics of the evolution of the innovation system have common patterns [29]. Interest in constructing mathematical models of innovation diffusion and using them to forecast the development of an innovation system emerged back in the 1960s, when pioneering works by Mansfield [30], Rogers [31] and Bass [32; 33] appeared. Every year, the number of studies devoted to the use of these models for solving specific economic problems increases, as does the number of reviews devoted to the analysis of the features of mathematical models of innovation systems and comparison of the areas of their application [29; 34-37, etc.].

Recent research focuses on the specifics of constructing nonlinear dynamic models for their subsequent use in solving problems of forecasting the evolution of innovation systems. This paper proposes a mathematical model of innovation diffusion, developed using the fundamental principles of nonlinear control theory.

These features include:

- a radical change in the behavioral goals of synthesized economic systems;
- direct consideration of the natural properties of nonlinear objects in synthesis procedures;
- formation of a new mechanism for generating feedback.

In the synergetic approach to the synthesis of economic control systems, the purpose of the functioning of a closed nonlinear system, in contrast to the classical theory of optimal control, is not only to meet the requirements put forward for the nature of the transition process, but, first of all, to ensure the desired asymptotic behavior of the original system on the attractor. This is because the behavior of any nonlinear dissipative system can be divided into a transitional motion phase, when its trajectories tend toward an attractor, and a phase of asymptotic motion along the desired attractor – the system's goal. This approach allows us to fundamentally resolve the problem of analytically synthesizing objective control laws for nonlinear objects.

However, it should be taken into account that a serious obstacle to mastering the above methods and, accordingly, introducing them into the practice of managing economic processes can be the well-known ideological and psychological problem associated with the famous triad of «nonlinearity – multidimensionality – multi-connectedness», which inspires horror in experienced specialists in the field of economic forecasting and management. The reason for this phenomenon is that these specialists are educated on the reductionist linear doctrine that underlies classical science. The same applies fully to classical and modern management theories, where the traditional linear approach dominates. It should be noted that applying this linear methodology to the management of promising dynamic economic entities is a profound ideological misconception.

Results and discussion. All of the above applies to a sufficient extent to many economic processes with quadratic nonlinearities, the dynamics of which are determined by so-called logistic curves. The work [38] provides a large list of phenomena in the field of science and technology that are modeled with a high degree of adequacy using the logistic equation. Here, the first-order logistic differential equation appears as a universal tool for describing economic reality, with the help of which it is possible to demonstrate the patterns of an evolving economy for forecasting purposes.

The process of evolution of an innovative product in its simplest form can be described by a logistic curve in the time domain, determined by ordinary differential equations of the form:

$$\frac{dx}{dt}=\gamma(x-a)(b-x), \qquad (1)$$

where t is a parameter expressing the time expenditure of economic agents on the development of an innovative product;

$x = x(t)$ is the total (cumulative) number of people who have already purchased an innovative product at the current point in time t (in the terminology Roger used when constructing the innovation diffusion model, this indicator was called «adopters»);

γ is a scale parameter whose inverse value is the characteristic time of the transition process in the model under consideration, $\gamma > 0$;

a and b are constants that limit from below and above, respectively, the characteristic significant indicator $x = x(t)$ of the functioning of an innovative product, $0 < a < b$.

The constant a, which corresponds to the lower limit of the values of the indicator $x = x(t)$, reflects the lower limit of the possibilities of dissemination of the innovative technology under consideration, while the constant b is the technological limit (or potential) of this innovative technology and demonstrates its maximum level of dissemination.

With an increase in the costs of developing and improving a given innovative technology and/or innovative product (regardless of the method of measuring these costs), the characteristic significant indicator of this innovation can only increase, therefore the function $x = x(t)$ increases smoothly throughout the entire area of its definition. The fact itself is that, according to the internal logic of equation (1), the growth rate of the quantity $x(t)$ is directly proportional to the distance of this quantity from its starting capabilities, i.e., the quantity $(x - a)$ shows that the greater this distance, the faster $x = x(t)$ grows. On the other hand, it follows from equation (1) that the growth of the value of $x(t)$ is proportional to the value of $(b - x)$, which means that the growth of $x = x(t)$ slows down as this indicator approaches its upper technological limit. It can be argued that the differential equation (1) is a mathematical formalization of the action of the law of mutual transition of quantitative changes into qualitative ones as applied to cumulative processes in the economy.

Equation (1) has a general solution that can be written explicitly as follows:

$$x(t) = a + \frac{(b-a)\cdot f(t)}{f(t)+C}, \qquad (2)$$

where $f(t) = \exp((b-a)\cdot\gamma\cdot t)$, C is arbitrary constant.

If equation (1) is supplemented with the initial condition $x(0) = x_0$, then the value of an arbitrary constant is found from the relation:

$$t = 0: \quad x_0 = a + \frac{(b-a)}{1+C} \Rightarrow C = \frac{b - x_0}{x_0 - a}. \qquad (3)$$

Moreover, the differential equation (1) has two special solutions $x_1^* = a$ and $x_2^* = b$, which cannot be obtained from the general solution for any values of the arbitrary constant. With respect to special solutions that are equilibrium positions of the innovation system, we note that $x_1^* = a$ is an unstable equilibrium, while $x_2^* = b$, on the contrary, is a stable equilibrium position. The economic meaning of these statements is clear if we proceed from the previously presented description of parameters a and b of the original differential equation (1). Thus, formula (2) with the corresponding comments (3) provides an exhaustive description of the phenomenon of self-organization of the process of innovation diffusion.

Now let's consider a situation where it is necessary to exert control over the dynamics of the innovation process. The necessity of this step can be explained by many reasons. For example, we may be dissatisfied with the equilibrium values of the process being studied and want to change them in the desired direction using some external influence. In this case, the goal of the synthesized system is to achieve the corresponding desired attractor – a new equilibrium position. The substantive justification for introducing control influence into the self-organization process in an economic system may be either entrepreneurial initiative on the part of the producer of an innovative product or the need to improve the effectiveness of government measures to regulate the diffusion of innovations. Formally, this control will be implemented as an additional component introduced into the basic differential equation (1) to describe the negative feedback loop. In this case, for $\gamma = 1$(which does not violate generality), we obtain a modified equation in the form:

$$\frac{dx}{dt} = \gamma(x-a)(b-x) - u, \qquad (4)$$

where $u = \varphi(x)$ is nonlinear control action.

We will assume that the function $\varphi(x)$ is quadratic in the variable x and has the following explicit representation:

$$\varphi(x) = \varphi_0 + \varphi_1 x + \varphi_2 x^2, \qquad (5)$$

where φ_0 and φ_1 are positive numbers, and the sign of φ_2 can be arbitrary.

We deliberately assume that we can change the sign of φ_2. This allows us to consider deviations from the linear equation in both directions. After substituting relation (5) into differential equation (4), we represent this differential equation as follows:

$$\frac{dx}{dt} = -(\varphi_0 + ab) + (a + b - \varphi_1)\cdot x - (\varphi_2 + 1)\cdot x^2. \qquad (6)$$

It is obvious that the structure of the singular points (equilibrium positions) of equation (6) is determined by solving the quadratic equation:

$$(\varphi_2 + 1)\cdot x^2 + (\varphi_1 - a - b)\cdot x + \varphi_0 + ab = 0. \qquad (7)$$

This quadratic equation (7) has the discriminant:

$$D = (\varphi_1 - a - b)^2 - 4\cdot(\varphi_2 + 1)(\varphi_0 + ab). \qquad (8)$$

From relation (8) it follows that for differential equation (6) there are two equilibrium positions (special points) if the condition is satisfied $D > 0$. If the discriminant's sign changes to the opposite, i.e., at $D < 0$, the quadratic equation (7) has no real roots. Furthermore, one double root may appear at $D = 0$, and accordingly, the two equilibrium positions merge into one.

Let us assume that the inequality $\varphi_2 + 1 > 0$ holds, i.e., $\varphi_2 > -1$. Then, by analogy with the differential equation (1), it can be shown that in equation (6), the larger value (parameter b) corresponds to a stable equilibrium position, and the smaller value (parameter a) corresponds to an unstable one. In this case, the relationship $0 < a < b$ is maintained. It should be emphasized that in a situation where the distance between equilibrium positions is small, the behavior of the system under study becomes complex. In fact, structural instability will occur, with the emergence of a saddle-node bifurcation.

To carry out a specific analysis of the topological properties of this bifurcation, we transform equation (6) to the corresponding normal form. The theory of normal forms, developed by Poincaré and Birkhoff, is one of the most powerful tools in the study of nonlinear dynamic systems and is useful for stability analysis and the classification of bifurcations [39]. To reduce equation (6) to normal form, we use a linear replacement of the original variable:

$$x(t)=\frac{2y(t)+a+b-\varphi_1}{2(\varphi_2+1)}. \qquad (9)$$

After performing the transformations, we obtain a differential equation (6) in the following form:

$$\frac{dy}{dt}=\mu-y^2, \qquad (10)$$

where

$$\mu=\frac{\varphi_0+ab}{\varphi_2+1}-\frac{1}{4}\cdot\left(\frac{\varphi_1-a-b}{\varphi_2+1}\right)^2. \qquad (11)$$

The differential equation (11) is the normal form of a saddle-node bifurcation («fold») provided that the value of μ is infinitely small [40-41]. If we transform expression (11), then taking into account (8), we obtain:

$$\mu=\frac{-D}{4(\varphi_2+1)^2}. \qquad (12)$$

From relation (12), it follows that equation (10) describes the behavior of model (6) in a small neighborhood of a two-fold equilibrium position. Thus, for $\mu>0$, the system described by equation (10) has two equilibria. Branch $y=\sqrt{\mu}$ corresponds to stable equilibrium (a node). If the system is disturbed slightly from this state, it will return. Branch $y=\sqrt{\mu}$ is always unstable (a saddle).

Any deviation from this point will cause the system to move away from it. The unstable equilibrium position is the boundary of attraction of the stable equilibrium.

At $\mu=0$, the two equilibrium positions merge into a single «semi-stable» equilibrium – a non-rough system emerges. At $\mu<0$, both equilibrium positions disappear. In this case, the system is in a dynamic state (continuously changing).

Let's follow the evolution of an innovative system near a stable equilibrium. As the coefficient φ_2 approaches its critical value at $\mu=0$, the region of attraction of this equilibrium on one side decreases, and after the equilibrium disappears, all solutions leave the phase region under consideration. In economic interpretation, this phenomenon is called a breakdown of equilibrium with a catastrophic loss of stability. The type of catastrophe observed is called a «fold.» Note that in catastrophe theory, a saddle-node bifurcation is called a «fold», while the term «saddle-node bifurcation» is usually used in physics and mathematics.

The behavior of phase trajectories for equation (10) depending on the sign of the parameter μ was simulated using MS Excel. The calculation results are presented in Fig. 1.

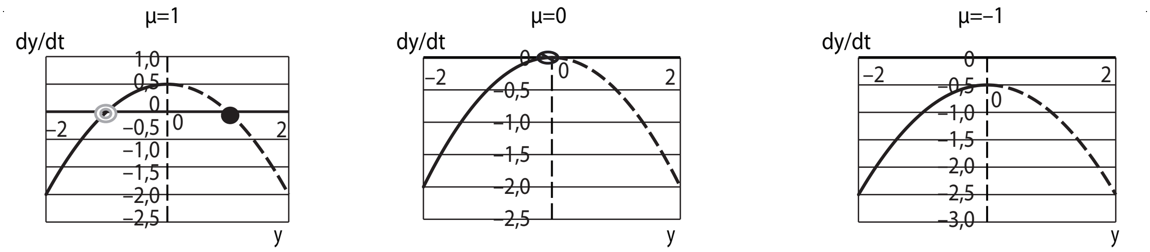

Fig. 1. Phase trajectories of the innovation system depending on the sign of the parameter μ

Source: as a result of the authors' calculations

In accordance with equation (10), the behavior of the phase trajectories of the evolution of the innovation system was modeled depending on the sign of the parameter μ using MS Excel. The calculation results are presented in Fig. 1. As can be seen from Fig. 1, for $\mu>0$, in particular, for $\mu=1$, there are two equilibrium positions. The one corresponding to $y<0$ is unstable. And the one corresponding to $y>0$ is stable. For $\mu=0$, both equilibrium positions merge into one, and for $\mu<0$, in particular, for $\mu=-1$, there are no equilibrium points.

In the case where $\varphi_2+1<0$, i.e., $\varphi_2<-1$, it is easy to show that a transcritical bifurcation occurs, with two branches with different tangents. This is the so-called «stability exchange» bifurcation. If we assume that the conditions $\varphi_0=-ab$ and $\varphi_1=a+b$ are satisfied, then differential equation (6) takes the form:

$$\frac{dx}{dt}=-(\varphi_2+1)\cdot x^2. \qquad (13)$$

Differential equation (13) is a typical example of describing an innovative system with positive feedback and can describe a peaking regime. The solution to equation (13) can be written in the following form:

$$x(t)=\frac{x_0}{1+x_0\cdot(\varphi_2+1)}. \qquad (14)$$

Taking into account that $\varphi_2+1<0$, one can calculate the «moment of aggravation» itself, i.e., when the solution (14) will tend to infinity:

$$t^*=\frac{-1}{x_0\cdot(\varphi_2+1)}. \qquad (15)$$

It is obvious that a solution (14) can exist only if $t<t^*$. In this case, the moment of exacerbation itself depends on the initial value of x_0 inversely proportional. This means that the larger the value of x_0, the shorter the lifetime of the solution. This is explaine-a real process of innovation diffusion over a given time can be described by equations that have solutions that increase in the peaking mode. In such a case, forecasting

the course of the process based on linear or more complex interpolation is doomed to failure. That is, it should not be considered possible to plan for tomorrow based on the results already achieved, since there is no reason to expect that the logic of the evolution of the innovation system will continue the same trends tomorrow as those observed today.

Regarding the solutions of the differential equation (10), we note that there are analytical solutions that depend on the sign of the parameter μ. We obtain:

$$\mu > 0: \quad y(t) = \frac{y_0\sqrt{\mu} + \mu \cdot \tanh(\sqrt{\mu} \cdot t)}{\sqrt{\mu} - y_0 \cdot \tanh(\sqrt{\mu} \cdot t)}; \qquad (16)$$

$$\mu = 0: \quad y(t) = \frac{y_0}{1 - y_0 \cdot t}; \qquad (17)$$

$$\mu < 0: \quad y(t) = \frac{y_0\sqrt{-\mu} + \mu \cdot tg(\sqrt{-\mu} \cdot t)}{\sqrt{-\mu} - y_0 \cdot tg(\sqrt{-\mu} \cdot t)}. \qquad (18)$$

From the relations (16)-(18) it is clear that changing the sign of the parameter μ implements the transition from the hyperbolic tangent to the trigonometric tangent through the hyperbola.

Thus, the self-organization process in the system under consideration results in the formation of attractors, to which the phase trajectories of the innovation system's evolution are drawn. This behavior of the system raises the question of the direction of the processes, i.e., their goals. These attractors have a dimension that is always smaller than the dimension of the original system, which means «forgetting» the initial conditions, i.e., the state from which movement toward the attractor begins. This results in the formation of invariant solutions to nonlinear differential equations, and these solutions are asymptotic. In our case of the evolution of an innovative system, the attractors can be the growth limits of the number of consumers of innovative products. This growth limit can be either natural or synthesized through external control.

Each attractor has its own basin of attraction, and therefore a boundary can be identified separating these basins. A sufficiently small change in the initial conditions near this boundary can then lead to qualitatively different behavior of the entire nonlinear system. This means that by applying sufficiently small influences to the system that correlate with its internal properties, it is possible to achieve qualitatively new behavior in a nonlinear innovation system far from its equilibrium position. This property, explained by the self-organization effect in dissipative systems, opens up opportunities for solving problems of managing nonlinear processes of innovation diffusion.

Conclusions. To summarize, we can conclude that the presence of a nonlinear control effect on the dynamics of the innovation process fundamentally changes the behavioral properties of the system under study, making it structurally unstable. This circumstance triggers the emergence of undesirable bifurcations, accompanied by a catastrophic loss of stability. Therefore, the most important task of synthesizing control laws is to prevent negative trends in the evolution of the innovation process during the implementation of an economic strategy for sustainable development.

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